Warm-up!

Awkward Question: Do y'all remember how to factor? No judgement.

$a(x) = x$

$(x-3)(x-2)$

$(2x+3)(3x-1)$

$(3x-5)(3x+5)$

$(x^2-4)(x^2+4)$

$(x-2)(x+2)(x^2+4)$

$(x^2)^2 - 7(x^2) + 12$

$(x^2-3)(x^2-4)$

$(x^2-3)(x-2)(x+2)$

$(x^2-1)(x^2-1)$

$(x-1)^2(x+1)^2$

$x^2(x-2) \quad -4(x-2)$

$(x-2)(x^2-4)$

$(x+2)(x-2)\ (x-2)$

$3(5x^2+3x-2)$

$3(5x-2)(x+1)$

$3(x+1)(5x-2)$

$5x^2+3x-2$

$5x^2+5x-2x-2$

$5x(x+1) \quad -2(x+1)$

$(x+1)(5x-2)$

-10

$5 \quad -2$

Unit 1 Day 4 (1.5): Fundamental

- Theorem of Algebra
- Finding Zeros
- Conjugate Theorems

Homework: Polynomial Zeros

Quiz Coming Up: Day 7

Awkward Question: Do y'all remember why we factor?

$a(x) = x^2 - 5x + 6$

$b(x) = 6x^2 + 7x - 3$

$c(x) = 9x^2 - 25$

$d(x) = x^4 - 16$

$e(x) = x^4 - 7x^2 + 12$

$f(x) = x^4 - 2x^2 + 1$

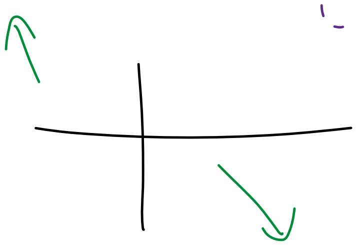
$g(x) = x^3 - 2x^2 - 4x + 8$

$h(x) = 15x^2 + 9x - 6$

5. Let $f(x) = (3x^2 + 4)(3 - 2x)^3(2x^2 - 4x)^3$. Determine the end behavior of f as x decreases without bound. Express your answer using the mathematical notation of a limit.

$$f(x) = (3x^2+4)(3-2x)(3-2x)(3-2x)(2x^2-4x)(2x^2-4x)(2x^2-4x)$$

$$-\square x^{11} + \_\_\_\_\_\_\_\_\_\_\_\_$$



$$\lim_{x\to-\infty} f(x) = \infty$$

$$\lim_{x\to\infty} f(x) = -\infty$$

4th degree

8. What is the least possible degree of a polynomial that would fit the data below?

x	-3	-2	-1	0	1	2	3
$f(x)$	8	37	40	41	40	13	-88

+29 +3 +1 −1 −27 −101

−26 −2 −2 −26 −74

+24 +0 −24 −48

−24 −24 −24

7. Below are values for $g(x)$, a cubic polynomial. Find the value of $g(11)$.

x	-4	-1	2	5	8	11	14
$g(x)$	-10	1	2	-1	-2	5	26

$+11$ $+1$ -3 -1 $+7$ $+21$

-10 -4 $+2$ $+8$ $+14$

$+6$ $+6$ $+6$ $+6$

In case I've made it this far and haven't said it yet:
Factoring is not a main point of focus for us. It's a tool we'll use in other problems. So that we can focus on the new concepts and not be caught up in the weeds of Algebra II, the factoring in this class might feel easier than Algebra II factoring.

Two Baby Facts and a Key Fact

1. Remainder Theorem: If a polynomial $f(x)$ is divided by $(x-k)$, then then the remainder is $f(k)$.

2. A polynomial $g(x)$ is a factor of $f(x)$ if $\frac{f(x)}{g(x)}$ has a remainder of 0.

3. Factor Theorem: $f(c)=0 \Leftrightarrow (x-c)$ is a factor of $f(x)$

$f(x) = (x+3)(x^3+7)$

$x=-3$

$x = 17$

$y = (x-17)(\_\ \_\ \_\ \_)$

Is $g(x) = (x + 7)$ a factor of $f(x) = x^2 - 9$?

$f(-7) = (-7)^2 - 9 = 40$

$(x-5)(1x^2-2x-3)$

Is $g(x) = (x-5)$ a factor of $f(x) = x^3 - 7x^2 + 7x + 15$?

$f(5) = (5)^3 - 7(5)^2 + 7(5) + 15$

$125 - 7(25) + 7(5) + 15$

$125 - 175 + 35 + 15$

$-50 + 50$

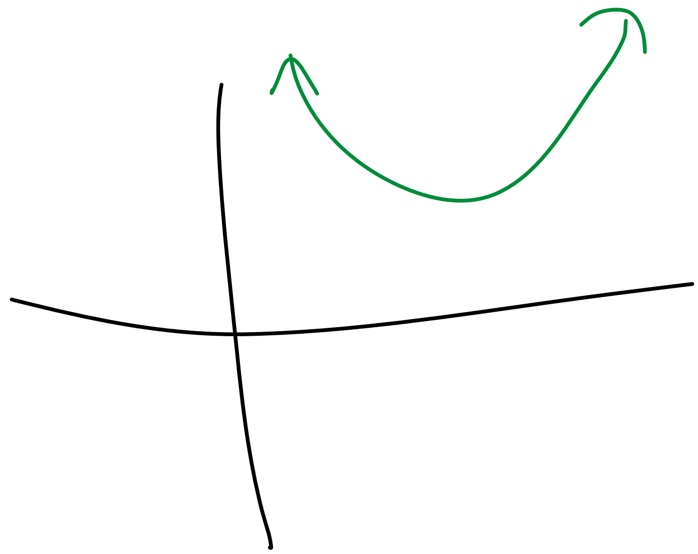
$f(5) = 0$

$$
\begin{array}{r|rrrr}
5 & 1 & -7 & 7 & 15 \\
 & & 5 & -10 & -15 \\
\hline
 & 1 & -2 & -3 & 0
\end{array}
$$

$1x^2 - 2x - 3$

Fundamental Theorem of Algebra

An nth degree polynomial has exactly n zeros.



Example 1 (time permitting)

A) Write a polynomial that has the following zeros: $-3, 0, 1$

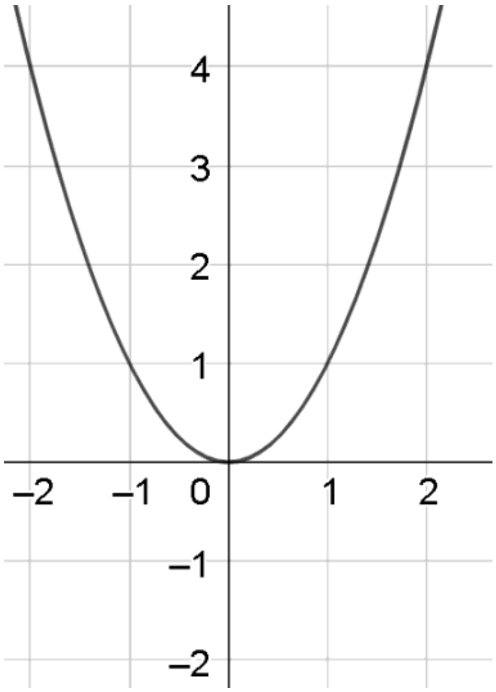
B) Write a polynomial that has the following zeros: $2, 2, 5$

C) Write a polynomial whose graph has the following x-intercepts: $(-2,0)$, $(0,0)$, $(4,0)$

D) Write a 4th degree polynomial whose graph has the following x-intercepts:
$(-2,0)$, $(0,0)$, $(4,0)$

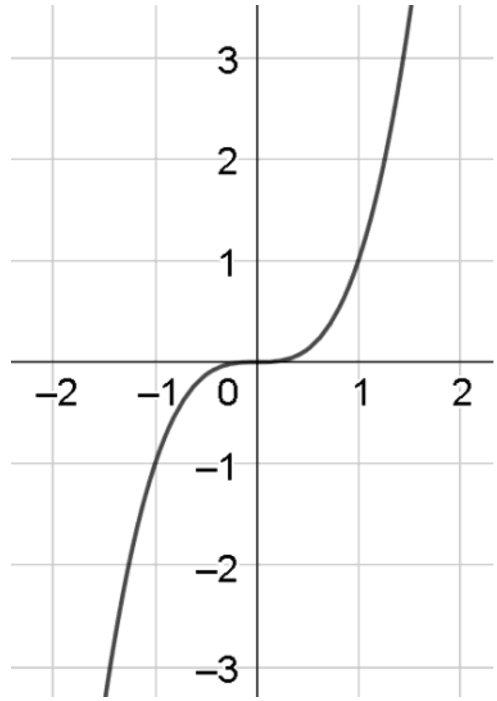
Tangent vs Crossing x-intercepts

An Example of Tangent:



"tangent to the x-axis"

An Example of Crossing:



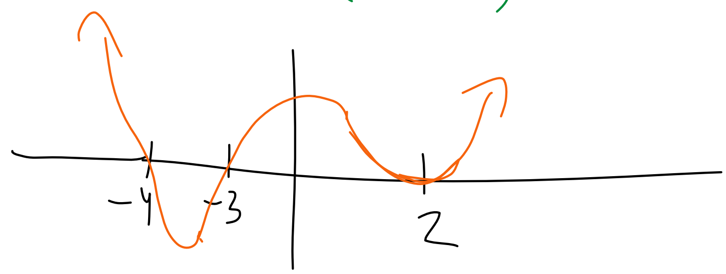
The Rule for Tangent vs Crossing

A graph crosses the x-axis at a zero if the multiplicity of that zero is odd.

A graph is tangent to the x-axis (i.e. "bounces" off the x-axis) at a zero if the multiplicity of that zero is even

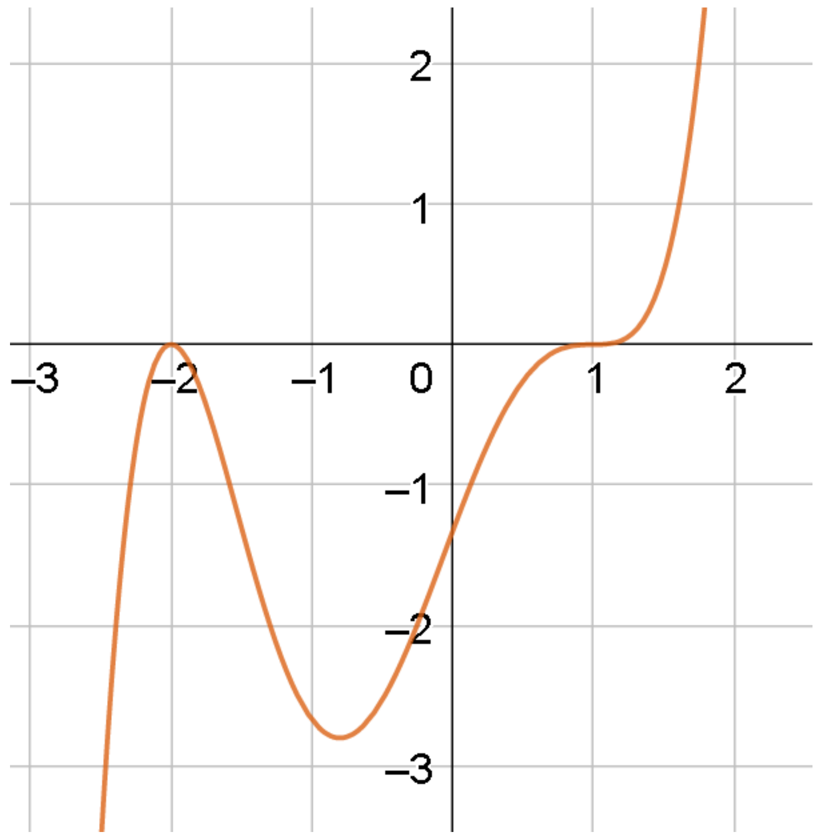
$f(x) = (x-2)(x-2)(x+3)(x+4)(x+4)(x+4)$

$x = 2$ (mult 2)
$x = -3$ (mult 1)
$x = -4$ (mult 3)



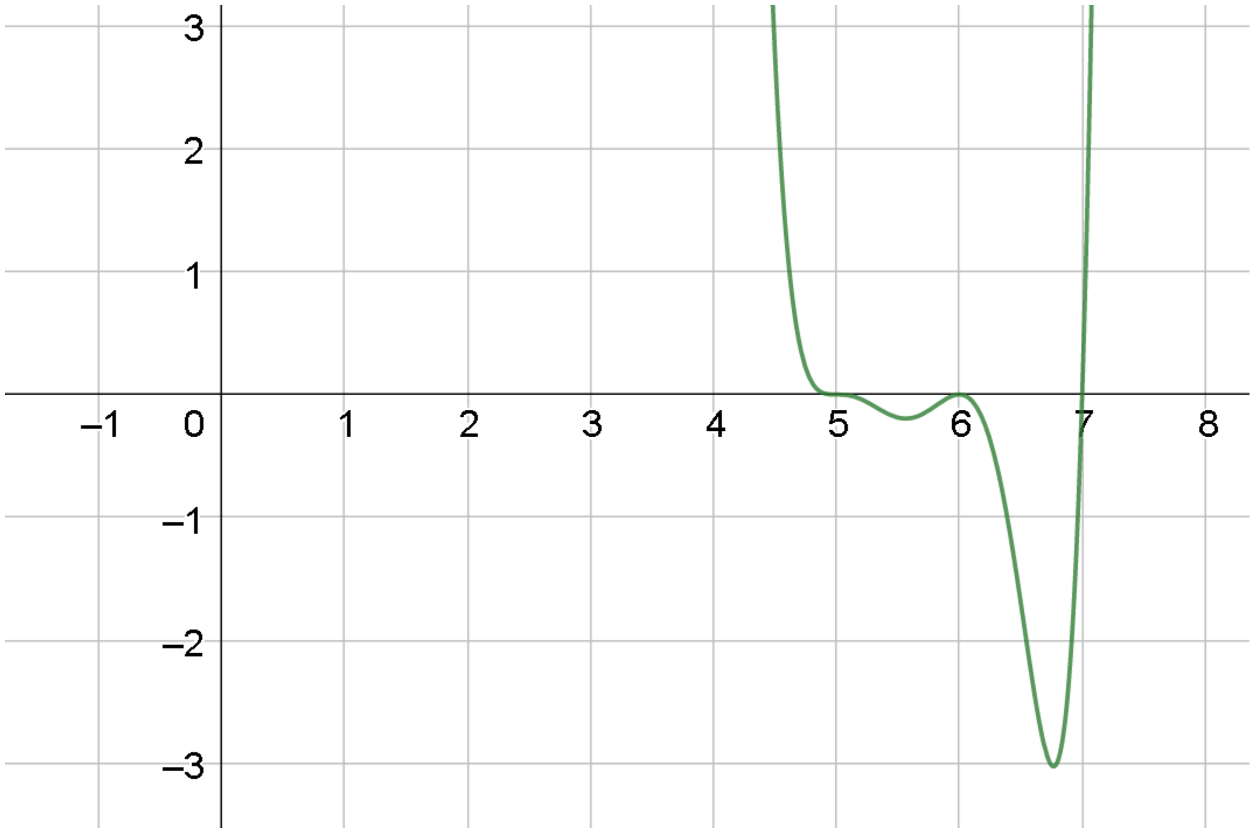
Example

$$f(x) = \frac{1}{3}(x+2)^2(x-1)^3$$



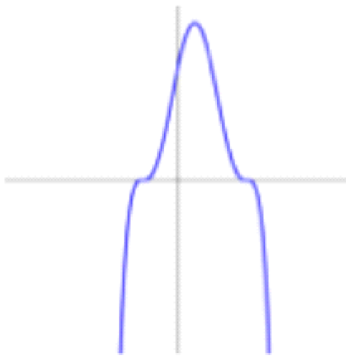
Another Example

$$g(x) = 4(x-5)^3(x-6)^2(x-7)$$

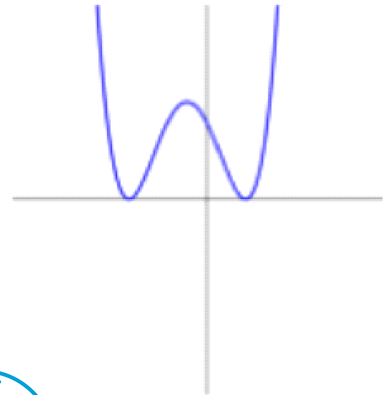


Which of the following graphs could represent the polynomial function $f(x) = a(x-b)^2(x-c)^3$

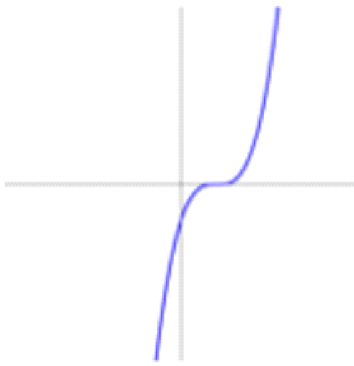
a)



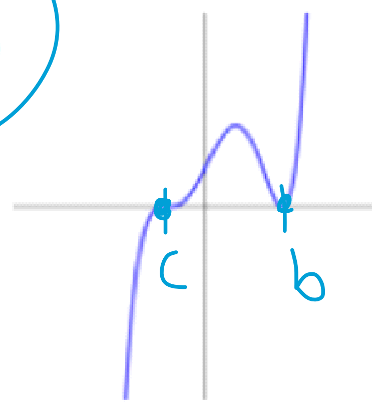
c)



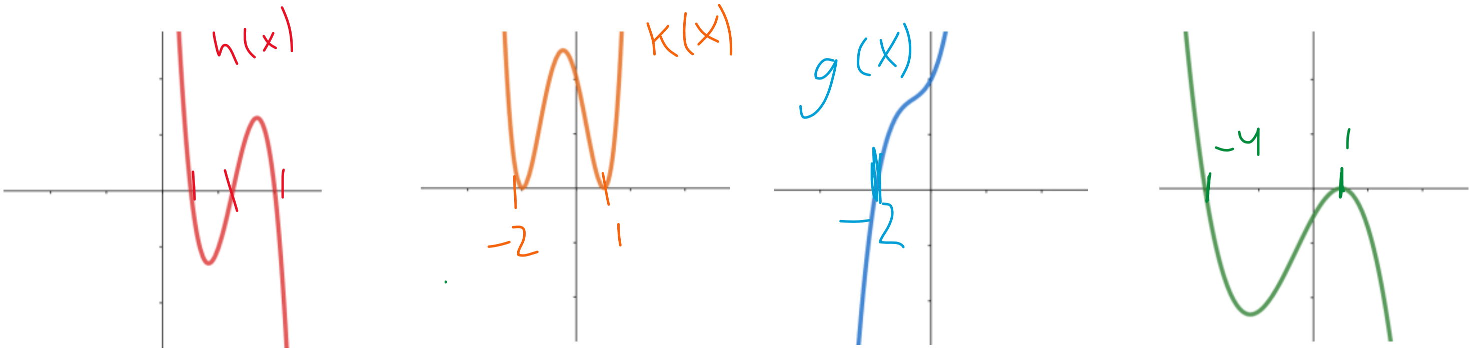
b)



d)



Factor each function completely. Then, match the graph to the function



$f(x) = -\frac{1}{4}(x-1)(x^2+3x-4)$

$-\frac{1}{4}(x-1)(x+4)(x-1)$

$g(x) = x^3+2x^2+2x+4$

$x^2(x+2) + 2(x+2)$

$(x+2)(x^2+2)$

$x^2+2=0$

$h(x) = (4-x)(2x^2-7x+5)$

$(4-x)(2x-5)(x-1)$

$x=4 \quad x=\frac{5}{2} \quad x=1$

$k(x) = (x^2+x-2)^2$

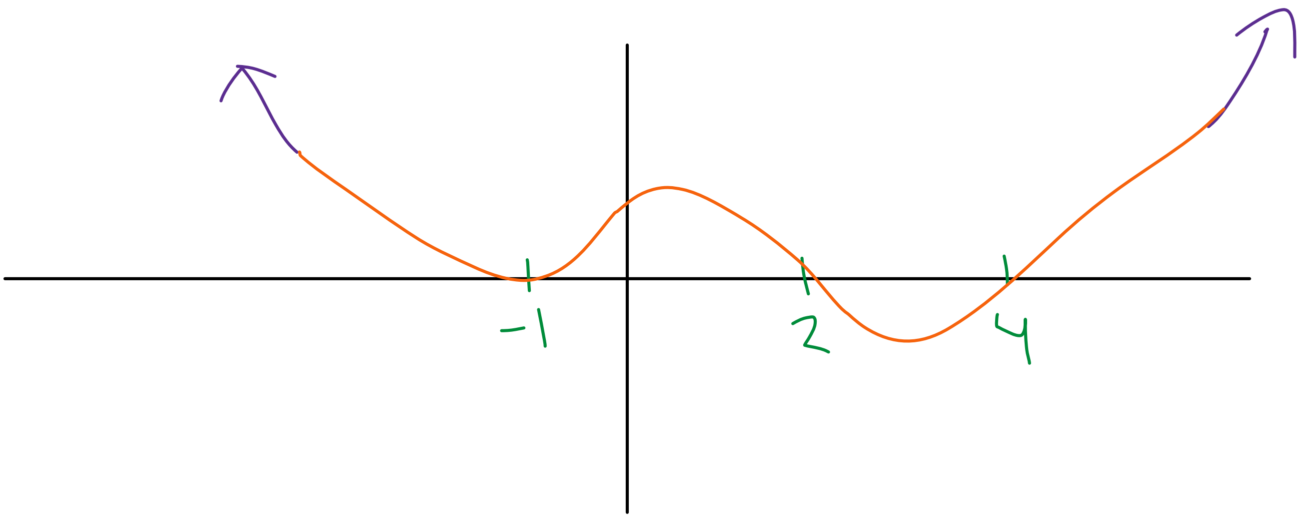
$((x+2)(x-1))^2$

$(x+2)^2(x-1)^2$

Sketch

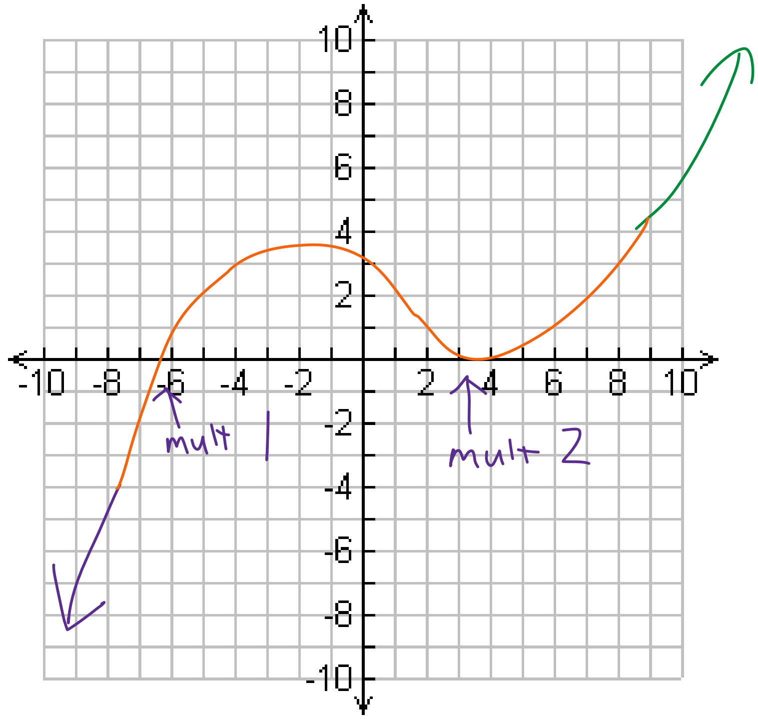
$\frac{1}{5}(x+1)(x+1)(x-2)(x-4) = \frac{1}{5}x^4 + \_ \_ \_ \_$

$$f(x) = \frac{1}{5}(x+1)^2(x-2)(x-4)$$



Sketch a polynomial with the given characteristics

- $\lim_{x \to -\infty} f(x) = -\infty$
- $\lim_{x \to \infty} f(x) = \infty$
- 1 negative crossing zero
- 1 positive "bouncing" zero



What is the smallest possible degree of this polynomial? 3

Could it be another degree?

could be degree 7

Complex Conjugate Theorem + Irrational Conjugate Theorem

Non-real zeros of a polynomial always come in... conjugate pairs

If a polynomial has rational coefficients, then irrational zeros always come in...

Examples:

If $2 - i$ is a root, then __$2+i$__ is also a root.

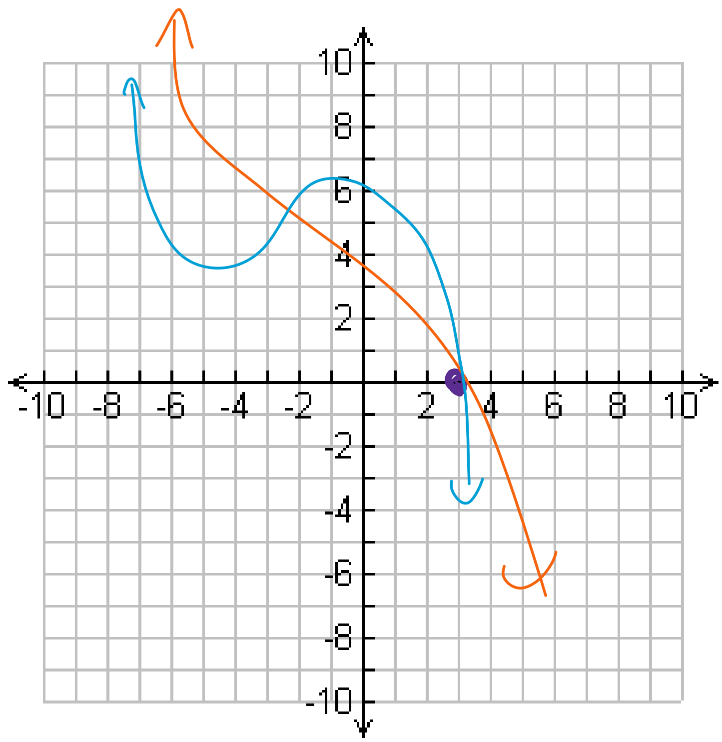
If $5 + 3i$ is a root, then __$5-3i$__ is also a root.

If $10i$ is a root, then __$-10i$__ is also a root.

If $2i - 6$ is a root, then __$-2i-6$__ is also a root.

Sketch a polynomial with the given characteristics

- Cubic polynomial
- $f(3) = 0$
- $f(2 - i) = 0$ $\quad f(2+i)=0$
- $\lim_{x \to -\infty} f(x) = \infty$



Is it possible for this function to have more than one x-intercept?

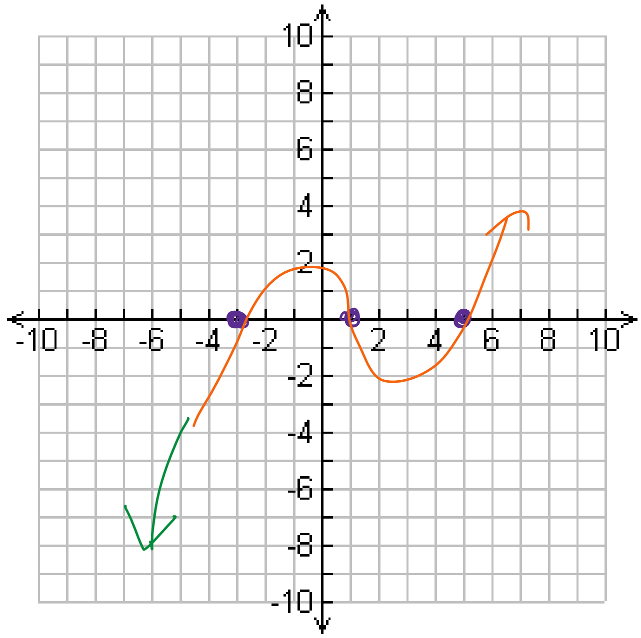
No

Is it possible for this function to have neither a relative max nor a relative min?

Yes

Sketch a polynomial with the given characteristics

- $x = -3, 1,$ and 5 are the only real zeros
- Fifth degree polynomial (aka "quintic")
- $\lim_{x \to -\infty} f(x) = -\infty$



This function must have at least _2_ local extrema

This function could have _2_ nonreal zeros.

Or it may have _0_ nonreal zeros.

An important theorem

Between every two distinct real zeros of a nonconstant polynomial function, there must be at least one real input-value corresponding to a local maximum or local minimum.